LA-UR-95-4527

First Order Quark-Gluon/Hadron Transition May Affect Cosmological Nucleosynthesis

L. Burakovsky*

Theoretical Division, T-8
Los Alamos National Laboratory
Los Alamos, NM 87545, USA

Abstract

In the model of a first order quark-gluon/hadron phase transition in which the hadronic phase is considered as vacuum bubbles growing in the quark-gluon background with chiral symmetry broken inside the bubble, we find the estimate for the length scale associated with inhomogeneities originated during the transition, 10 m $\lesssim \ell \lesssim$ 40 m, being sufficient to produce significant effects on cosmological nucleosynthesis.

Key words: nucleosynthesis, nucleation, inhomogeneities, surface tension, quark-gluon/hadron transition

PACS: 12.40.Aa, 64.60.Q, 68.10.C, 97.10.C, 98.80.Ft

Recent lattice gauge simulations indicate that the pure $SU(3)$ gauge theory has a first order phase transition between the low temperature color confining hadronic phase and the high temperature deconfined plasma phase [1]. Because of the first order

*Bitnet: BURAKOV@QCD.LANL.GOV.

nature of the phase transition, these two phases can coexist at the critical temperature with a distinct phase interface in between; this interface carries a positive amount of extra free energy, i.e. a surface tension (σ).

The surface tension is a fundamental parameter in the description of the time evolution of hot hadronic matter as it proceeds through the phase transition. In the scenario of a homogeneous nucleation [2] the probability for a bubble to nucleate is

$$p(T) = \exp\left\{-\frac{16\pi\sigma^3}{3T_c(p-p^{'})^2}\right\}. \tag{1}$$

In this model, the bubble at nucleation time has a radius large enough to be hydrostatically stable in the supercooled plasma, and to sustain a difference in pressure $(p-p^{'})$ between two phases.

The question has been raised in the literature [3, 4] whether baryon inhomogeneities could originate during a first order quark-gluon/hadron phase transition that would alter the successful conventional calculations of cosmological nucleosynthesis [5]. An essential ingredient in these calculations is the surface tension for bubbles of the new hadronic phase. The length scale associated with the inhomogeneities has been given in ref. [4] [1] (we will arrive at this relation below):

$$\ell \simeq (3.4\times 10^4\ \mathrm{m})(\sigma/\mathrm{MeV}^3)^{3/2}(T_c/\mathrm{MeV})^{-13/2}, \tag{2}$$

where T_c is the critical temperature for a first order quark-gluon/hadron phase transition. Campbell *et al.* [7] have obtained an estimate for this surface tension in the framework of the effective low energy Lagrangian for broken chiral and scale invariance [8] in the hadronic phase,

$$\sigma \simeq (70\ \mathrm{MeV})^3, \tag{3}$$

which is in good agreement with the estimate made in ref. [9]. Recently the surface tension has been calculated in quenched QCD by lattice numerical methods [6, 10, 11], as well as analytically in the mean-field approximation [12, 13]. When performed on $N_t = 2$ lattices, the numerical studies have produced the results [6]

$$\sigma/T^3 = 0.24(6) \tag{4}$$

and [10]

$$\sigma/T^3 = 0.12(2), \tag{5}$$

while for $N_t = 4$ lattice [11]

$$\sigma/T^3 = 0.027(4). \tag{6}$$

With σ given by (3)-(6) and $T_c \simeq 100$ K, the formula (2) gives the estimate $\ell \simeq 0.5-1$ m, which is hardly of interest for nucleosynthesis since significant effects on

[1] Eq. (2) takes into account the correction given in ref. [6].

nucleosynthesis require $\ell > 10$ m and probably $\ell > 100$ m.[2] Because of a strong dependence of T_c, ℓ will be smaller for larger T_c; e.g., for $T_c \simeq 150$ K, Eq. (2) with σ given by (3) yields $\ell \simeq 0.1$ m.

Bhattacharya *et al.* [13] have found the expression for the interface tension between distinct $Z(N)$ vacua above the deconfinement transition for a pure $SU(N)$ gauge theory ($N = 2, 3$),

$$\sigma = \frac{4(N-1)\pi^2}{3\sqrt{3N}} \frac{T^3}{g}, \tag{7}$$

where g is the gauge coupling constant. For a typical value of $g \simeq 1-2$ (corresponding to $\alpha_s \equiv g^2/4\pi \simeq 0.1 - 0.3$) at temperature of the order of a typical deconfinement one, $T \simeq 150$ MeV [15], the formula (2) yields the estimate

$$\ell \simeq (15 - 30) \text{ m}, \tag{8}$$

which is considerably larger than that obtained from Eqs. (3)-(6) above.

In this letter we show that the estimate for ℓ following from the values of the surface tension obtained in refs. [6, 7, 9, 10, 11], $\ell \simeq 0.5 - 1$ m, should be raised by one-two order of magnitude, in agreement with Eq. (8), if one considers the new hadronic phase as vacuum bubbles growing in the quark-gluon environment, with chiral symmetry broken (i.e., nonzero value of the quark condensate) inside the bubble.

At temperatures well below T_c, hadronic matter consists of the lightest hadrons, the pions. At T near T_c, the $\pi\pi$ interaction becomes strong, the $\pi\pi$ amplitudes at the relevant energies become so large that they allow for some bound states, the resonances [16]. The method for taking into account such resonance interaction was suggested by Belenky and Landau [17] as considering the unstable particles on an equal footing with the stable ones in the thermodynamic quantities, by means of a resonance spectrum. Such a spectrum in both the statistical bootstrap model [18, 19] and the dual resonance model [20] takes on the form

$$\rho(m) \sim m^a \, e^{m/T_0}, \tag{9}$$

where a and T_0 are constants. This treatment of a hadronic resonance gas leads to a singularity in the thermodynamic functions at $T = T_0$ [18, 19] and, in particular, to an infinite number of the effective degrees of freedom in the hadronic phase, thus making a transition to the quark-gluon phase impossible. To cope with this difficulty, it is normally suggested that the hadrons are strongly interacting particles with finite range interactions among them which increase with increasing density. These interactions cannot be ignored in discussing the thermodynamics of hadronic matter; in fact, it has been shown [21] that the neglect of such interactions leads to unphysical

[2] As shown in ref. [14], $\ell \simeq 30$ m is needed for the most interesting models of a non-homogeneous universe.

behavior for a deconfinement phase transition mentioned above. (The mass spectrum (9) originates from the clustering of hadrons [19]; such a clustering corresponds to an effective attractive interaction and would not allow for a transition to the quark-gluon phase since for this transition a repulsive interaction at small distances is necessary.) In ref. [7] the hadronic resonance spectrum (9) was modified to include an effective repulsive interaction by means of i) a hard-core potential, ii) an excluded volume (Van der Waals approach). It has been shown that in both cases the number of the effective degrees of freedom in the hadronic phase is now finite (namely, 6.5-7, as read off from Figs. 6,7 of ref. [7]), and a normal (first order) transition to the quark-gluon phase becomes possible. For our present purposes we shall restrict ourselves to the pions alone (i.e., to the 3 effective degrees of freedom), and think of the hadronic phase as that of the pions which are brought to the boiling point at T_c with the energy density $\rho_\pi(T_c) = 3\pi^2/90\; T_c^4$.

The usual scheme of a first order phase transition is as follows. With the increase of energy deposition (e.g., in relativistic heavy ion collisions), bubbles of the quark-gluon plasma "vapor" are formed until at $\rho \sim g/3\;\rho_\pi(T_c)$ the system is entirely vapor (g being the total number of degrees of freedom in the quark-gluon phase, which is 37 for two-flavor plasma and 47.5 for three-flavor one). The quark-gluon to hadron transition in the early universe proceeds in a slightly different way: when the universe is about 10 μsec old, its temperature drops down to $T_c \simeq 150$ MeV. However, the phase transition does not start immediately but the interface tension causes supercooling by the amount of $\triangle T/T_c \simeq 0.02(\sigma/T_c^3)^{3/2}$ [22]. After the period of supercooling the hadron bubbles start nucleating; this process gives rise to shock waves which reheat the universe back to T_c, thus hindering the further formation of the hadron bubbles [22] (in a different scenario, bubble formation and growth is sufficient to begin reheating the system due to the release of latent heat [23]). The transition continues because of the growth of previously created bubbles, until the system gets entirely hadronized. The average distance between the bubbles turns out to be [4]

$$\ell \simeq 10^{-4}\; R_H(\sigma/T_c^3)^{3/2}\;\mathrm{m}, \tag{10}$$

where R_H is the horizon size at the transition time,

$$R_H \sim \frac{3\cdot 10^8\;\mathrm{m}}{(T_c/\mathrm{MeV})^2} \sim 10\;\mathrm{km}. \tag{11}$$

In view of (10),(11), one obtains the estimate (2) for the scale of inhomogeneities in the hadron distribution which may have affected cosmological nucleosynthesis.

The transition temperature is calculated by equating pressures. If the quark-gluon plasma is treated within the bag model, then

$$p_\pi = 3\frac{\pi^2}{90}T_c^4 = p_{QGP} = g\frac{\pi^2}{90}T_c^4 - B \tag{12}$$

(B being the bag constant, $B \simeq (245 - 255 \text{ MeV})^4$ [24]), giving the value of $T_c \simeq 170$ MeV for two-flavor plasma and $\simeq 160$ MeV for three-flavor one.

In conventional bag model the difference between the quark condensate values on different sides of a bag surface is normally disregarded. To estimate the effect of the quark condensate discontinuity on the properties of hadrons, we treat the letter within the bag model and use the energy-momentum tensor $T_q^{\mu\nu}$ of the quark fields $\psi_f^a(x)$ with massless quarks inside the bag. In zero order approximation of a power expansion in the QCD coupling constant α_s the following expression holds:

$$T_q^{\mu\nu} = \frac{i}{2} \sum_f \left(\bar{\psi}_f^a \gamma^\mu \partial^\nu \psi_f^a - (\partial^\nu \bar{\psi}_f^a) \gamma^\mu \psi_f^a \right), \tag{13}$$

where $a = 1, 2, 3$ is $\mathrm{SU}(3)_c$ color index, and $f = u, d, s$ is $\mathrm{SU}(3)_f$ flavor index. The linear boundary condition[3] [25]

$$i\gamma^\mu n_\mu \psi_f^a(x) = \psi_f^a(x) \tag{14}$$

on the bag surface with external normal n_μ corresponds to the relation $\bar{\psi}_f^a \psi_f^a = 0$ on the interior side of the bag surface. On the exterior side $\bar{\psi}_f^a \psi_f^a \neq 0$ because the corresponding vacuum condensates have nonzero values [26]. In this case the quark field contributions to the bag energy density $\rho_q = T_q^{00}$ contain not only conventional part corresponding to the quark kinetic energy but also contain the usually ignored surface part. The surface part arises from the contribution of the discontinuity of the quark condensate values on both sides of a bag surface [27]:

$$E_s = -\frac{1}{4} S \sum_f \langle \bar{\psi}_f^a \psi_f^a \rangle = \sigma_{vac} S. \tag{15}$$

Here S is a surface area of the bag, and

$$\sigma_{vac} \equiv -\frac{1}{4} \sum_f \langle \bar{\psi}_f^a \psi_f^a \rangle. \tag{16}$$

Typical "empirical" values discussed in the literature are [26, 28]

$$\langle \bar{\psi}_u^a \psi_u^a \rangle \simeq \langle \bar{\psi}_d^a \psi_d^a \rangle \simeq -(240 \pm 25 \text{ MeV})^3, \tag{17}$$

$\langle \bar{\psi}_s^a \psi_s^a \rangle$ being of similar magnitude. The commonly adopted value of the quark condensate for calculations within the framework of QCD sum rules is [29]

$$\langle \bar{\psi}^a \psi^a \rangle = -0.0241 \text{ GeV}^3 \cong -(289 \text{ MeV})^3. \tag{18}$$

[3]In the chiral bag model, the boundary condition reads $i\gamma^\mu n_\mu \psi_f^a(x) = e^{i\tau^b \pi^b \gamma^5} \psi_f^a(x)$, where $b = 1, 2, 3$ is the isospin index, and couples the quark fields on one side of the bag boundary with the pion field on the other side in a highly nonlinear manner. We do not discuss this model here.

Sometimes people consider even higher values of the quark condensate, e.g. [30]

$$\langle\bar{\psi}_u^a\psi_u^a\rangle = \langle\bar{\psi}_d^a\psi_d^a\rangle = -(287\ \mathrm{MeV})^3,$$
$$\langle\bar{\psi}_s^a\psi_s^a\rangle = -(306\ \mathrm{MeV})^3, \tag{19}$$

or [31]

$$\langle\bar{\psi}^a\psi^a\rangle \simeq -0.032\ \mathrm{GeV}^3 \cong -(315\ \mathrm{MeV})^3. \tag{20}$$

We shall take

$$\langle\bar{\psi}_u^a\psi_u^a\rangle \simeq \langle\bar{\psi}_d^a\psi_d^a\rangle \simeq \langle\bar{\psi}_s^a\psi_s^a\rangle \simeq -(270-280\ \mathrm{MeV})^3, \tag{21}$$

which lies somehow between the values provided by (17)-(20). Then the value of σ_{vac} calculated from Eq. (16) is

$$\sigma_{vac} \simeq (245-255\ \mathrm{MeV})^3. \tag{22}$$

This surface tension is strong in comparison with the vacuum pressure (the bag constant) [4] [24]

$$B = -\rho_{vac} = \frac{9}{32}\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{a\mu\nu}\rangle - \frac{1}{4}\sum_f m_f\langle\bar{\psi}_f^a\psi_f^a\rangle \simeq (245-255\ \mathrm{MeV})^4. \tag{23}$$

Here $G_{\mu\nu}^a$ is a gluon field stress tensor and m_f is a typical bare (current) quark mass.

Now consider the hadronic phase arising as a result of a first order quark-gluon/hadron transition in the form of vacuum bubbles growing in the bulk quark-gluon plasma. Such a vacuum bubble is a sperical surface with quarks and gluons outside it obeying the bag boundary conditions[5] (14). Inside the bubble we have nonzero quark $\langle\bar{\psi}_f^a(x)\psi_f^a(x)\rangle$ and gluon $\langle G_{\mu\nu}^a G^{a\mu\nu}\rangle$ condensates. Therefore, one can apply arguments similar to the previous ones and obtain the same expression for the interface surface tension as above (Eq. (16)). Thus, the value of σ given by Eq. (22) should be used as the realistic value for the bubble surface tension in the expression for the length scale associated with the inhomogeneities, Eq. (2). The exect value for the critical temperature of a quark-gluon/hadron phase transition is not known. In the framework of chiral perturbation theory, Gerber and Leutwyler [34] have calculated the temperature of a chiral symmetry restoration transition (to be associated with the hadron to quark-gluon one) $T_c \simeq 170$ MeV in the $\mathrm{SU}(2)_f$ chiral limit. The value of the critical temperature provided by lattice gauge simulations is currently [15] $T_c \simeq 140$ MeV. We, therefore, consider the value of the critical temperature of a quark-gluon/hadron transition to be in the temperature interval

$$140\ \mathrm{MeV} \lesssim T_c \lesssim 170\ \mathrm{MeV}, \tag{24}$$

[4]The estimate that we obtained, $\sigma^{1/3} \approx B^{1/4}$, was also suggested in ref. [32].
[5]The bag boundary conditions for gluons are discussed in ref. [33].

as granted by both chiral perturbation theory and the most recent lattice simulations (note that both values of T_c estimated above within the bag model lie in this temperature range). For the temperature interval (24) and the value of the bubble tension given by (22), Eq. (2) gives

$$10 \text{ m} \lesssim \ell \lesssim 40 \text{ m}, \tag{25}$$

which is one-two order of magnitude as much as the estimate following from the values of σ found in refs. [6, 7, 9, 10, 11]. In view of the aforementioned estimate $\ell > 10$ (or 100) m, this value of the length scale should be considered as being sufficient to produce significant effects on nucleosynthesis. Note that the estimates of refs. [13, 14] lie in the interval (25).

By further adjustment of the values of σ and T_c the estimate (25) may be raised or diminished, respectively. For example, for $T_c \simeq 100$ MeV and σ given by (22) one obtains $\ell \simeq 200 - 250$ m.

The question has been raised in ref. [35] whether inhomogeneities originating during a first order quark-gluon/hadron transition can produce cosmological beryllium or boron. In contrast to simple inhomogeneities like those with $\ell \simeq 0.5 - 1$ m, inhomogeneities associated with the length scale (25) are likely to produce cosmological Be or B. If this is really the case, it will have great implications for Big Bang nucleosynthesis. Cosmological Be or B are a signature for significant density variations [35]. Such variations could lead to planetary mass black holes [36] or quark nuggets [37, 38] that could serve as the dark matter of the universe.

As remarked in [1], another interesting point which may lead to a non-standard scenario of nucleosynthesis is that there may be a baryon number contrast in the hadron and the quark-gluon plasma phases. The thermodynamic advantage to place most of the baryon number into the quark-gluon phase was analyzed in ref. [14]. A large penalty in the free energy is paid when a unit of baryon number is placed in the hadron phase because of the large mass of the baryons, as compared to the phase of massless quarks for which there is no mass penalty in the free energy. This may not be the case if an effective baryon mass is temperature dependent and drops significanfty as T gets closer to T_c, as suggested in a series of papers by Brown and Rho [39]. It is most probably that the estimate of ref. [14] for the baryon number contrast in two phases, $R \sim O(100)$, should be diminished by one order of magnitude. Even in this case, the baryon number contrast in two phases will still remain large.

Two phenomena, the large inhomogeneities scale in the hadron distribution and the large baryon number contrast in the hadron and the quark-gluon plasma phases should lead to sizable effects on the standard scenario of cosmological nucleosynthesis. With the observations made in this letter, conventional calculations of cosmological nucleosynthesis should be amended by nontrivial details. New work on this and related subjects is in progress.

Acknowledgements

I wish to thank T. Bhattacharya, L.P. Horwitz and E. Mottola for very valuable discussions.

References

[1] For a review, see B. Petersson, Nucl. Phys. A **525** (1991) 237c

[2] L.D. Landau and E.M. Lifshitz, *Statistical Physics,* 3rd ed., (Pergamon Press, Oxford, 1980)

[3] J.H. Applegate and C.J. Hogan, Phys. Rev. D **31** (1985) 3037
J.H. Applegate, C.J. Hogan and R.J. Scherrer, Phys. Rev. D **35** (1987) 1151; Astrophys. J. **329** (1988) 572
C.R. Alcock, G.M. Fuller and G.J. Mathews, Astrophys. J. **320** (1987) 439
W.A. Fowler and R.A. Malaney, Astrophys. J. **333** (1988) 14
H. Kurki-Suonio and R.A. Matzner, Phys. Rev. D **39** (1989) 1046
H. Kurki-Suonio, R.A. Matzner, K.A. Olive and D.N. Schramm, Astrophys. J. **353** (1990) 406
K. Rummukainen, Nucl. Phys. A **525** (1991) 569c
K. Jedamzik, G.M. Fuller and G.J. Mathews, Astrophys. J. **423** (1994) 50

[4] G.M. Fuller, G.J. Mathews and C.R. Alcock, Phys. Rev. D **37** (1988) 1380

[5] J. Yang *et al.,* Astrophys. J. **281** (1984) 493
K.A. Olive, D.N. Schramm, G. Steigman and T. Walker, Phys. Lett. B **236** (1990) 454
T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive and H.S. Kang, Astrophys. J. **376** (1991) 51
M.S. Smith, L.H. Kawano and R.A. Malaney, Astrophys. J. Suppl. **85** (1993) 219
C.J. Copi, D.N. Schramm and M.S. Turner, Science **267** (1995) 192

[6] K. Kajantie, L. Karkkainen and K. Rummukainen, Nucl. Phys. B **333** (1990) 100

[7] B.A. Campbell, J. Ellis and K.A. Olive, Nucl. Phys. B **345** (1990) 57

[8] D.J. Gross, R. Pisarski and L. Yaffe, Rev. Mod. Phys. **53** (1981) 43

[9] C. Alcock and E. Farhi, Phys. Rev. D **32** (1985) 1273

[10] S. Huang, J. Potvin, C. Rebbi and S. Sanielevici, Phys. Rev. D **42** (1990) 2864; **43** (1991) 2056

[11] R. Brower, S. Huang, J. Potvin and C. Rebbi, Phys. Rev. D **46** (1992) 2703

[12] Z. Frei and A. Patkós, Phys. Lett. B **222** (1989) 469

[13] T. Bhattacharya, A. Gocksch, C. Korthals Altes and R.D. Pisarski, Phys. Rev. Lett. **66** (1991) 998

[14] C. Alcock, G.M. Fuller, G.J. Mathews and B. Mayer, Nucl. Phys. A **498** (1989) 301c

[15] C. Bernard *et al.,* Phys. Rev. D **45** (1992) 3854

[16] E.V. Shuryak, *The QCD Vacuum, Hadrons and the Superdense Matter,* (World Scientific, Singapore, 1988)

[17] S.Z. Belenky and L.D. Landau, Sov. Phys. Uspekhi **56** (1955) 309, Nuovo Cim. Suppl. **3** (1956) 15

[18] R. Hagedorn, Nuovo Cim. **35** (1965) 216; Nuovo Cim. Suppl. **6** (1968) 169, 311

[19] S. Frautschi, Phys. Rev. D **3** (1971) 2821

[20] S. Fubini and G. Veneziano, Nuovo Cim. A **64** (1969) 811

[21] J. Cleymans, K. Redlich, H. Satz and E. Suhonen, Z. Phys. C **33** (1986) 151

[22] T. DeGrand and K. Kajantie, Phys. Lett. B **147** (1984) 273
K.Kajantie and H. Kurki-Suonio, Phys. Rev. D **34** (1986) 1719

[23] L.P. Csernai and J.I. Kapusta, Phys. Rev. Lett. **69** (1992) 737

[24] G.E. Brown, H.A. Bethe and P.M. Pizzochero, Phys. Lett. B **263** (1991) 337

[25] A. Chodos *et al.,* Phys. Rev. D **9** (1974) 3471

[26] M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B **147** (1979) 385, 448

[27] Yu.E. Pokrovskii, Sov. J. Nucl. Phys. **50** (1989) 565

[28] L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. **127** (1985) 1

[29] M. Asakawa and C.M. Ko, Nucl. Phys. A **560** (1993) 399

[30] M. Lutz, S. Klimt and W. Weise, Nucl. Phys. A **542** (1992) 521

[31] P. Zhuang, J. Hüfner and S.P. Klevansky, Nucl. Phys. A **576** (1994) 525

[32] E. Farhi and R.L. Jaffe, Phys. Rev. D **30** (1984) 2379

[33] I. Mardor and B. Svetitsky, Phys. Rev. D **44** (1991) 878

[34] P. Gerber and H. Leutwyler, Nucl. Phys. B **321** (1989) 387

[35] D.N. Schramm, B. Fields and D. Thomas, Nucl. Phys. A **544** (1992) 267c

[36] M. Crawford and D.N. Schramm, Nature **298** (1982) 538

[37] A.R. Bodmer, Phys. Rev. D **4** (1971) 1601
C. Alcock and A. Olinto, Ann. Rev. Nucl. Part. Sci. **38** (1988) 161

[38] E. Witten, Phys. Rev. D **30** (1984) 272

[39] G.E. Brown and M. Rho, Phys. Lett. B **222** (1989) 324, **237** (1990) 3;
Phys. Rev. Lett. **66** (1991) 2720
G.E. Brown, M. Rho and M. Soyeur, Nucl. Phys. A **553** (1993) 705c